

TEACHER GUIDE

HMH Guide to
Success
in Math
for the **SAT**[®]

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Contents

The math section of the SAT[®] test assesses a variety of skills that are organized into four domains. Each domain is divided into several content dimensions, and each content dimension has a number of skills associated with it. The purpose of this publication is to provide practice on these skills using the two question types on the SAT[®] test: multiple choice and grid-in. For each skill, there is a two-page practice test that includes sample questions with worked-out solutions as well as practice questions. An answer sheet for each practice test is provided for students to record their answers. An answer key page, providing answers to the questions on the practice test, follows each answer sheet.

The skills listed in the Table of Contents below and on the practice tests are paraphrases of the wording of the skills identified in the College Board publication *Test Specifications for the Redesigned SAT[®]* (pages 137–145).

Heart of Algebra

Linear equations in one variable

| | |
|--|----|
| Use linear equations in one variable to solve problems in a variety of contexts. | 1 |
| Model a real-world situation with a linear equation in one variable, solve the equation, and interpret the solution. | 5 |
| Use the structure of a linear equation in one variable to solve the equation efficiently. | 9 |
| Interpret a linear equation in one variable in context, and define the conditions affecting the number of solutions. | 13 |
| Proficiently solve a variety of linear equations in one variable. | 17 |

Linear functions

| | |
|--|----|
| Write and use linear functions to solve problems representing real-world situations. | 21 |
| Determine a linear function that models a relationship between two quantities. | 25 |
| Calculate the inputs and/or outputs for a linear function for a real-world situation, and interpret the meaning. | 29 |
| Connect various representations of linear functions by deriving one from the other. | 33 |
| Use two input/output pairs or one input/output pair and the rate of change to find the rule for a linear function. | 37 |

Linear equations in two variables

| | |
|--|----|
| Write and use a linear equation in two variables to solve problems representing real-world situations. | 41 |
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| Write a linear equation in two variables to model a relationship between two quantities. | 45 |
| For a linear equation in two variables, interpret the parts of the equation and its solution based on the context. | 49 |
| Connect various representations of linear equations in two variables by deriving one from the other. | 53 |
| Find an equation for a line given two points, a point and the slope, or a point and a perpendicular or parallel line. | 57 |
| Systems of two linear equations in two variables | |
| Write and use a system of two linear equations in two variables to solve problems in real-world situations. | 61 |
| Create a system of linear equations in two variables, and interpret solutions in terms of context when appropriate. | 65 |
| Connect various representations of systems of equations by deriving one from the other. | 69 |
| Utilize algebraic structure to efficiently solve a system of two linear equations in two variables. | 73 |
| Determine the number of solutions for a system, and use structure to interpret parts of systems of equations. | 77 |
| Proficiently solve a system of linear equations in two variables. | 81 |
| Linear inequalities in one or two variables | |
| Write and use linear inequalities in one or two variables to solve problems in real-world situations. | 85 |
| Create linear inequalities, and interpret the solution in terms of the context. | 89 |
| Interpret components of linear inequalities in one or two variables. | 93 |
| Connect various representations of linear inequalities in one or two variables by deriving one from the other. | 97 |
| Interpret a point in the solution set of a linear inequality or system of linear inequalities. | 101 |
| <u>Problem Solving and Data Analysis</u> | |
| Ratios, rates, proportional relationships, and units | |
| Find and use proportional relationships, ratios, rates, and units in real-world situations. | 105 |
| Solve problems involving derived units and unit conversion. | 109 |
| Use scale factors to compute quantities that are in a proportional relationship. | 113 |

Percentages

- Find and use percentages to solve problems in real-world situations. **117**
- Recognize and use the relationship between growth factor and percent change. **121**

One-variable data: distributions and measures of center and spread

- Choose appropriate graphical representations for data. **125**
- Interpret information about real-world data from a given representation. **129**
- Analyze and interpret data distributions represented in tables, histograms, and plots. **133**
- Calculate, compare, and interpret the mean, median, and range of quantitative data. Interpret standard deviation. **137**
- Use measures of center and spread to compare distributions. **141**
- Determine the effect of outliers on mean and median. **145**
- Determine the mean of an appropriate data set. **149**

Two-variable data: models and scatterplots

- Compare data values given in a scatterplot to values predicted by a model that fits the data. **153**
- Interpret characteristics of the line of best fit in real-world situations. **157**
- Use tables and graphs to understand relationships between quantities. **161**
- Interpret data in scatterplots and line graphs, and fit appropriate models. **165**
- Identify appropriate graphs, and interpret graphs in real-world situations. **169**
- Find a linear, quadratic, or exponential function that best fits a given set of data. **173**
- Distinguish between linear and exponential growth. **177**
- Estimate and use the line of best fit for a scatterplot. **181**

Probability and conditional probability

- Determine and interpret probability and conditional probability in simple real-world situations. **185**
- Use formulas for probability and conditional probability. **189**

Inference from sample statistics and margin of error

- Estimate population parameters using sample statistics. Utilize margin of error (without calculating it). **193**
- Interpret margin of error, and understand how it is affected by sample size. **197**

| | |
|--|-----|
| Evaluating statistical claims: observational studies and experiments | |
| Determine to which population the results of a random sample can be extended. | 201 |
| Determine if a study with random or non-random assignment provides evidence for a causal relationship. | 205 |
| Explain why there is evidence for a causal relationship. | 209 |
| Explain why results from a sample apply only within the population from which it was drawn. | 213 |

Passport to Advanced Math

Equivalent expressions

| | |
|--|-----|
| Use algebraic structure and the properties of operations to identify and write equivalent expressions. | 217 |
| Add, subtract, and multiply polynomials proficiently. | 221 |

Nonlinear equations in one variable and systems of equations in two variables

| | |
|--|-----|
| Solve a variety of nonlinear equations and systems of equations in two variables; recognize valid solutions. | 225 |
| Given a single-variable, nonlinear equation, interpret its parts or the solution based on the context. | 229 |
| Solve an equation or formula in two or more variables for a variable of interest. | 233 |
| Complete the square or use the quadratic formula to solve quadratic equations in standard form. | 237 |

Nonlinear functions

| | |
|--|-----|
| Use quadratic or exponential functions to solve real-world problems. | 241 |
| Create, use, and make connections between different representations of quadratic or exponential functions. | 245 |
| Work with factorable polynomial functions and simple rational functions, and their graphs. | 249 |

Additional Topics in Math

Area and volume

| | |
|---|-----|
| Solve problems about geometric figures using information such as length, surface area, or volume. | 253 |
|---|-----|

Lines, angles, and triangles

- Solve problems involving congruent and similar triangles using appropriate concepts and theorems. **257**
- Determine the statements needed to prove relationships or satisfy theorems. **261**
- Understand that the scale factor k changes side lengths but not angle measures. **265**
- Know and apply theorems about vertical angles, triangles, and parallel lines cut by a transversal. **269**

Right triangles and trigonometry

- Solve real-world problems using the Pythagorean theorem and right triangle trigonometry. **273**
- Determine values of sine, cosine, and tangent by using similarity of triangles. **277**
- Calculate the remaining values in a right triangle when given one side length and one acute angle measure. **281**
- Use the relationship between sine and cosine of complementary angles to solve problems. **285**
- Apply properties of special right triangles to find lengths and trigonometric ratios for 30° , 45° , and 60° angles. **289**

Circles

- Solve problems using definitions, properties, and theorems involving circles and parts of circles. **293**
- Use trigonometric ratios in the unit circle and radian measure to solve problems. **297**
- Represent a circle in the xy -plane with an equation. **301**
- Understand the relationship between equations and graphs of circles. **305**
- Understand the relationship between the standard form of the equation of a circle and its graph. **309**
- Convert angle measurements between degrees and radians. **313**
- Solve problems about circles in the xy -plane using the distance formula or by completing the square in x and y . **317**

Complex numbers

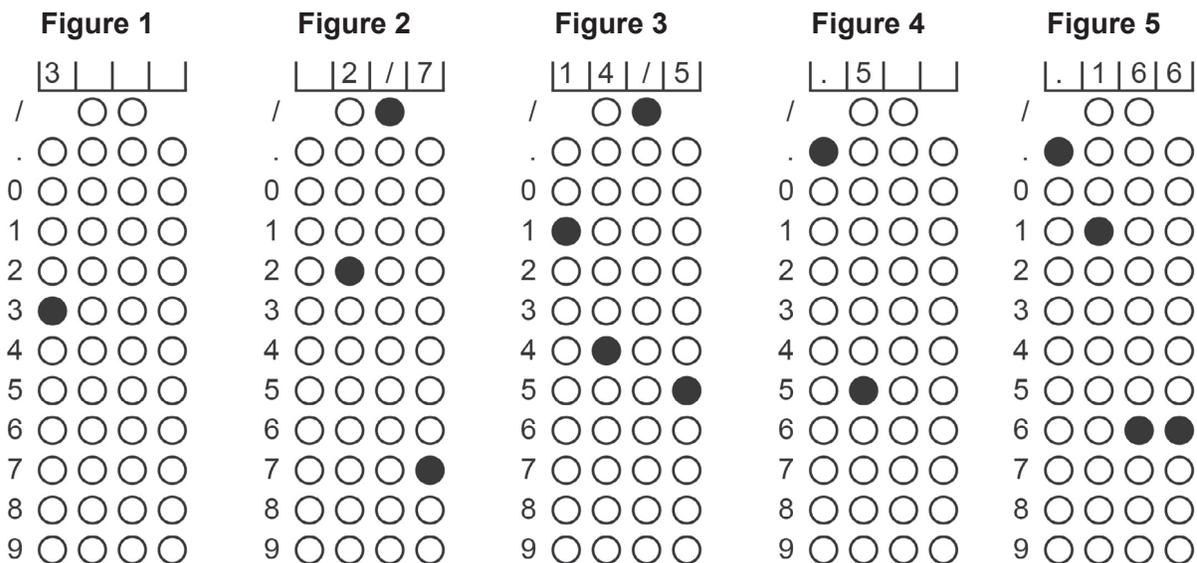
- Add, subtract, multiply, and divide complex numbers, and solve problems. **321**

Gridding In Answers

You should record your answers on the answer sheet provided for each practice test. For the grid-in portion of a test, each numerical answer needs to be entered in a grid. Though not required, writing your answer in the boxes provided at the top of the grid is recommended. Each grid includes four boxes, one above each of the four columns of circles. Write just one character in each box. Notice the characters (fraction bar, decimal point, digits) shown along the left side of the grid. Below each box, you should fill in the circle that is to the right of the character you have written in that box. When fewer than four boxes are needed, be sure to leave the unused columns blank.

Things to remember when gridding in numerical answers:

- 1) Credit for a question is given only if the correct circles have been filled in.
- 2) Mark only one circle in each column, completely filling that circle.
- 3) The grid does not support negative numbers, so no question has a negative answer.
- 4) For answers with fewer than four characters, the answer can start in any column. If your answer is 3, use any of the four columns; if your answer is $\frac{2}{7}$, use either the first three columns or the last three columns to grid the answer. [See Figures 1 & 2.]
- 5) A mixed number result must be gridded as an improper fraction or as its decimal equivalent. A result of $2\frac{4}{5}$ should be gridded as either $\frac{14}{5}$ or 2.8. [See Figure 3.]
- 6) Decimal numbers less than 1 must be gridded without a leading zero. If your result is 0.5, you must grid it as just .5 (without a 0 in the ones place). [See Figure 4.]
- 7) If your result is a decimal with more than 3 digits (and no rounding instructions were given), then fill the grid with four characters by either rounding the final digit or truncating the decimal at this digit. Both answers will be accepted. [See Figure 5; the answer .167 would also be accepted.]



Interpret a linear equation in one variable in context, and define the conditions affecting the number of solutions.

| SAMPLE MULTIPLE CHOICE | |
|---|--|
| <p>$-15 + 9(10 - 2x) = -3(6x) + \underline{\hspace{2cm}}$</p> <p>Fill in the missing term so that the equation has infinitely many solutions.</p> <p>A) 90 B) 75 C) $9x$ D) $-18x$</p> | <p>For there to be infinitely many solutions, the two sides of the equation must be identical. Simplify both sides of the equation.</p> $-15 + 9(10 - 2x) = -3(6x) + \underline{\hspace{2cm}}$ $-15 + 90 - 18x = -18x + \underline{\hspace{2cm}}$ $75 - 18x = -18x + \underline{\hspace{2cm}}$ <p>There is no need to simplify any further; it can be seen that the missing term is 75.</p> <p>The correct answer is B. A B C D <input type="radio"/> <input checked="" type="radio"/> <input type="radio"/> <input type="radio"/></p> |

1

$$75 + 3(\underline{\hspace{1cm}} + 18) = 14x + 7(4x + 9)$$

Which term, when inserted at the blank line, would result in an equation that has no solutions?

A) -43 C) $14x$
B) 14 D) $42x$

4

$$26.2 = 6.5h$$

The equation above gives the number of hours h it takes for a runner to finish a 26.2-mile marathon. What is the runner's speed, in miles per hour?

A) 0.25 C) 6.5
B) 4.03 D) 8.06

2

$$(x + 4) - 4(-5 - x) = \frac{1}{15}(75x) - 2$$

How many solutions does the equation above have?

A) Infinitely many solutions
B) A unique solution: $x = -4.1$
C) A unique solution: $x = 45$
D) No solutions

5

$$16 - h - 4^2 = -9(h + 4) - 2(-18 - 4h)$$

How many solutions does the equation above have?

A) No solutions
B) Infinitely many solutions
C) A unique solution: $h = 18$
D) A unique solution: $h = -2$

3

The expression $45 + 0.05t$ gives the total charges per month, in dollars, for a cellular plan where t is the total number of text messages sent during the month. Which of the following could be the total amount for a monthly bill?

A) \$43 C) \$60.24
B) \$52.60 D) \$77.61

6

To install new recessed lighting in a home, an electrician charges a trip fee plus a set amount per light. She uses the expression $25 + 275x$ to calculate the cost to install x new lights. Which of the following could be the total amount she charges a homeowner for a job?

A) \$3,050 C) \$1,730
B) \$2,335 D) \$275

| SAMPLE GRID-IN | |
|---|--|
| <p style="text-align: center;">$P = 2.4t + 314$</p> <p>The population P, in millions, of the United States is modeled by the equation shown above where t is the number of years since 2012. In what year does the model indicate that the population of the United States reached 326 million people?</p> | <p>For $P = 326$ (million), the equation becomes $326 = 2.4t + 314$. Solve this equation for t:</p> <p style="text-align: center;">$326 = 2.4t + 314$</p> <p style="text-align: center;">$12 = 2.4t$</p> <p style="text-align: center;">$5 = t$</p> <p>Add 5 years to the year 2012:</p> <p style="text-align: center;">$2012 + 5 = 2017$</p> <p>So, the model predicts that the U.S. population reached 326 million in 2017. Grid in 2017.</p> <p>Time-Saving Tip: The coefficient of the variable t means that every year the population increased by 2.4 million. Recognizing that the increase is $326 - 314 = 12$ (million), dividing 12 by 2.4 gives $t = 5$ years: $2012 + 5$, or 2017.</p> |

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7

$$11y - 5(4 - y) = 3(y - 17) + \underline{\hspace{2cm}}$$

For the equation above, give the value needed for the equation to have the unique solution $y = -2$.

8

$$12(8c - 1) + \underline{\hspace{2cm}} = 7(9c) + 11(3c + 2)$$

What constant is needed on the left side for the equation to have infinitely many solutions?

9

A chimney-cleaning company charges a trip fee plus an hourly rate for their service. The equation $30(2x + 1) = 420$ models the number of hours x spent on a recent job. What is the hourly rate, in dollars, charged by the company?

10

$$6(4 - z) + 2(3z + 10) = 7(z + \underline{\hspace{2cm}}) + z$$

For the equation above, give the value needed for the equation to have the unique solution $z = -5$.

11

$$60x + 738 = 5,238$$

The monthly cost to produce x chairs at a factory is modeled by the equation above. What are the fixed costs for operating the factory?

12

The equation $84 - 2.8t = 42$ models the amount of time t , in hours, it takes to pump half of the oil (in millions of gallons) from an oil tanker. What is the rate, in millions of gallons per hour, at which the pump removes the oil?

13

$$5(9w - 2) - (15w + 2) = 8$$

What value of w is a unique solution to this equation?

14

$$8v - (-5v - 6) = 9v + 1 + 4v + \underline{\hspace{2cm}}$$

In the equation above, what value added to the right side will create an equation with infinitely many solutions?

Interpret a linear equation in one variable in context, and define the conditions affecting the number of solutions.

ANSWER SHEET

1. A B C D

2. A B C D

3. A B C D

4. A B C D

5. A B C D

6. A B C D

7.

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Interpret a linear equation in one variable in context, and define the conditions affecting the number of solutions.

ANSWER KEY

1. C
2. D
3. B
4. C
5. B
6. A
7. 5
8. 34
9. 60
10. 12
11. 738
12. 2.8
13. $\frac{2}{3}$ or 0.666 or 0.667
14. 5

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A two-page practice test for each skill
in these four domains:

- Heart of Algebra (26 skills)
- Problem Solving and Data Analysis (28 skills)
- Passport to Advanced Math (9 skills)
- Additional Topics in Math (18 skills)

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